# Color Image Segmentation Using Acceptable Histogram Segmentation

Julie Delon[1], Agnes Desolneux[2], Jose Luis Lisani[3], and Ana Belen Petro[3]

[1] CMLA, ENS Cachan, France
julie.delon@cmla.ens-cachan.fr
[2] MAP5, Université Paris 5, France
desolneux@math-info.univ-paris5.fr
[3] Univ. Illes Balears, Spain
{joseluis.lisani,anabelen.petro}@uib.es

**Abstract.** In this paper, a new method for the segmentation of color images is presented. This method searches for an acceptable segmentation of 1D-histograms, according to a "monotone" hypothesis. The algorithm uses recurrence to localize all the modes in the histogram. The algorithm is applied on the hue, saturation and intensity histograms of the image. As a result, an optimal and accurately segmented image is obtained. In contrast to previous state of the art methods uses exclusively the image color histogram to perform segmentation and no spatial information at all.

## 1 Introduction

Image segmentation refers to partitioning an image into different regions that are homogenous with respect to some image feature. Thought most attention on this field has been focused on gray scale images, color is a powerful feature that can be used for image segmentation.

Among the classical techniques for color images segmentation, pixel-based techniques do not consider spatial information. The simplest pixel-based technique for segmentation is histogram thresholding which assumes that the histogram of an image can be separated into as many peaks (modes) as different regions are present in the image.

The existing techniques for histogram thresholding can be distinguished by the choice of the color component from which the histogram is obtained and by the modes extraction criterion. Concerning the first of these issues, some approaches ([7]) consider 3D histograms, that simultaneously contain all the color information in the image. However, storage and processing of multidimensional histograms is computationally expensive. For this reason most approaches consider 1D histograms computed for one or more color components in some color space (see for example [5] and [6]).

With respect to the modes extraction criterion, most methods are based on parametric approaches. That is, they assume the histogram to be composed of $k$ random variables of a given distribution, for instance the Gaussian distribution, with different averages and variances. However, these methods require an estimation of the number of modes in the final segmentation and, moreover, the found modes have not proven to be relevant.

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In this work, we present an automatic method for color image segmentation based on the detection of "menaingul modes" in histograms. We choose the HSI color space for the application of our approach to color images. This space has the advantage of separating color from intensity information.

The paper is structured as follows. The basic ideas of the method are exposed in the next section; in section 3, the application to color segmentation is presented; section 4 is devoted to display and comment some results on color image segmentation; the conclusions are presented in the final section.

## 2 Histogram Analysis by Helmholtz Principle

In 2003, A. Desolneux, L. Moisan and J.M. Morel ([3]) defined a new parameter-free method for the detection of meaningful events in data. An event is called $\varepsilon$-meaningful if its expectation under the a contrario random uniform assumption is less than $\varepsilon$. Let us state what this definiton yields in the case of the histogram modes.

### 2.1 Uniform Hypothesis. Meaningful Intervals and Meaningful Gaps of a Histogram

We will consider a discrete histogram $r$, that is $N$ points distributed on $L$ values, $\{1, ..., L\}$. For each discrete interval $[a, b]$ of $\{1, ..., L\}$, $r(a, b)$ will represent the proportion of points in the interval. For each interval $[a, b]$ of $\{1, ..., L\}$, we note $p(a, b) = \frac{b-a+1}{L}$ the relative length of the interval. The value $p(a, b)$ is also, under the uniform assumption, the probability for a point to be in $[a, b]$. Thus, the probability that $[a, b]$ contains at least a proportion $r(a, b)$ of points among the $N$ is given by the binomial tail $\mathcal{B}(N, Nr(a, b), p(a, b))$, where $\mathcal{B}(n, k, p) = \sum_{j=k}^{n} \binom{n}{j} p^j (1-p)^{n-j}$. The number of false alarms of $[a, b]$ is:

$$NFA([a, b]) = \frac{L(L+1)}{2} \mathcal{B}(N, Nr(a, b), p(a, b)).$$

Thus, an interval $[a, b]$ is said $\varepsilon$-meaningful if it contains "more points" than the expected average, in the sense that $NFA([a, b]) \leq \varepsilon$, that is

$$\mathcal{B}(N, Nr(a, b), p(a, b)) < \frac{2\varepsilon}{L(L+1)}.$$

In the same way, an interval $[a, b]$ is said to be an $\varepsilon$-meaningful gap if it contains "less points" than the expected average.

If $\varepsilon$ is not too large (in practice, we will always use $\varepsilon = 1$) an interval cannot be at the same time an $\varepsilon$-meaningful interval and an $\varepsilon$-meaningful gap. Now, these binomial expressions are not always easy to compute, especially when $N$ is large. In practice, we adopt the large deviation estimate to define meaningful intervals and gaps.

**Definition 1.** *The relative entropy of an interval* $[a, b]$ *(with respect to the prior uniform distribution p) is defined by*

$$H([a, b]) = r(a, b) \log \frac{r(a, b)}{p(a, b)} + (1 - r(a, b)) \log \frac{1 - r(a, b)}{1 - p(a, b)}.$$

$H([a,b])$ is the Kullback-Kleiber distance between two Bernoulli distributions of respective parameters $r(a,b)$ and $p(a,b)$ ([2]), that is $H([a,b]) = \mathrm{KL}(r(a,b)||p(a,b))$.

**Definition 2.** *An interval $[a,b]$ is said to be an **$\varepsilon$-meaningful interval** (resp. **$\varepsilon$-meaningful gap**) if $r(a,b) \geq p(a,b)$ (resp. $r(a,b) \leq p(a,b)$) and if*

$$H([a,b]) > \frac{1}{N} \log \frac{L(L+1)}{2\varepsilon}$$

### 2.2 Monotone Hypothesis

How can we detect meaningful intervals or gaps if we know that the observed objects follow a non-uniform distribution (e.g. decreasing or increasing)? We want now to define the meaningfulness of an interval with respect to the decreasing hypothesis (the definitions and results for the increasing hypothesis can be deduced by symmetry). We will call $\mathcal{D}(L)$ the space of all decreasing densities on $\{1, 2, ..., L\}$ and $\mathcal{P}(L)$ be the space of normalized probability distributions on $\{1, 2, ..., L\}$, *i.e.* the vectors $r = (r_1, ..., r_L)$ such that: $\forall i \in \{1, 2, ..., L\}$, $r_i \geq 0$ and $\sum_{i=1}^{L} r_i = 1$.

If $r \in \mathcal{P}(L)$ is the normalized histogram of our observations, we need to estimate the density $p \in \mathcal{D}(L)$ in regards to which the empirical distribution $r$ has the "less meaningful" gaps and intervals, which is summed up by the optimization problem

$$\tilde{r} = argmin_{p \in \mathcal{D}(L)} \min_{[a,b] \in \{1,2,...,L\}} KL(r(a,b)||p(a,b)).$$

The meaningfulness of intervals and gaps can then be defined relatively to this distribution $\tilde{r}$. Note that the uniform distribution is a particular case of decreasing density, which means that this formulation strengthens the previous theory: if there is no meaningful interval or gap in regards to the uniform hypothesis, there will be no meaningful interval or gap in regards to the decreasing hypothesis.

However, this optimization problem is uneasy to solve. We choose to slightly simplify it by approximating $\tilde{r}$ by the Grenander estimator $\bar{r}$ of $r$ ([4]), which is defined as the nonparametric maximum likelihood estimator restricted to decreasing densities on the line.

**Definition 3.** *The histogram $\bar{r}$ is the unique histogram which achieves the minimal Kullback-Leibler distance from $r$ to $\mathcal{D}(L)$, i.e. $KL(r||\overline{r}) = \min_{p \in \mathcal{D}(L)} KL(r||p)$.*

It has been proven ([1]) that $\overline{r}$ can easily be derived from $r$ by an algorithm called "Pool Adjacent Violators" that leads to a unique decreasing step function $\bar{r}$.

**Pool Adjacent Violators**
*Let $r = (r_1, ..., r_L) \in \mathcal{P}$ be a normalized histogram. We consider the operator $D : \mathcal{P} \to \mathcal{P}$ defined by: for $r \in \mathcal{P}$, and for each interval $[i,j]$ on which $r$ is increasing, i.e. $r_i \leq r_{i+1} \leq ... \leq r_j$ and $r_{i-1} > r_i$ and $r_{j+1} < r_j$, we set*

$$D(r)_k = \frac{r_i + ... + r_j}{j - i + 1} \quad \text{for} \quad k \in [i,j], \quad \text{and} \quad D(r)_k = r_k \quad \text{otherwise.}$$

*This operator $D$ replaces each increasing part of $r$ by a constant value (equal to the mean value on the interval).*

*After a finite number (less than the size $L$ of $r$) of iterations of $D$ we obtain a decreasing distribution denoted $\overline{r}$, $\overline{r} = D^L(r)$.*

An example of histogram and its Grenander estimator is shown on Figure 1.

Now, the definitions of meaningful interval gaps are analogous to the ones introduced in the uniform case, the uniform prior being just replaced by the global decreasing estimate $\overline{r}$ of the observed normalized histogram $r$.

**Definition 4.** *Let $r$ be a normalized histogram. We say that an interval $[a,b]$ is $\varepsilon$-meaningful for the decreasing hypothesis (resp. an $\varepsilon$-meaningful gap for the decreasing hypothesis) if $r(a,b) \geq \overline{r}(a,b)$ (resp. $r(a,b) \leq \overline{r}(a,b)$) and*

$$H_{\overline{r}}([a,b]) > \frac{1}{N} \log \frac{L(L+1)}{2\varepsilon},$$

*where $H_{\overline{r}}([a,b]) = \mathrm{KL}(r(a,b)||\overline{r}(a,b))$.*

We are now able to define precisely what we called "to be almost decreasing on a segment".

**Definition 5.** *We say that a histogram **follows the decreasing (resp. increasing) hypothesis** on an interval if it contains no meaningful gap for the decreasing (resp. increasing) hypothesis on the interval.*

## 2.3 Acceptable Segmentations

The aim of our segmentation is to split the histogram in separated "modes". We will call "mode" an interval on which the histogram follows the increasing hypothesis on a first part and the decreasing one on the second part.

**Definition 6.** *We say that a histogram $r$ **follows the unimodal hypothesis** on the interval $[a,b]$ if it exists $c \in [a,b]$ such that $r$ follows the increasing hypothesis on $[a,c]$ and $r$ follows the decreasing hypothesis on $[c,b]$.*

Such a segmentation exists. Indeed, the segmentation defined by all the minima of the histogram as separators follows obviously the unimodal hypothesis on each segment. But if there are small fluctuations it is clear that it is not a reasonable segmentation (see Fig. 2 left). We present a procedure that finds a segmentation much more reasonable than the segmentation defined by all the minima. We want to build a minimal (in terms of numbers of separators) segmentation, which leads us to introduce the notion of "acceptable segmentation".

**Definition 7.** *Let $r$ be a histogram on $\{1,...,L\}$. We will say that a segmentation $s$ of $r$ is **acceptable** if it verifies the following properties:*

- *$r$ follows the unimodal hypothesis on each interval $[s_i, s_{i+1}]$.*
- *there is no interval $[s_i, s_j]$ with $j > i+1$, on which $r$ follows the unimodal hypothesis.*

The two requirements allow us to avoid under-segmentations and over-segmentations, respectively. It is clear in the discrete case that such a segmentation exists: we can start with the limit segmentation containing all the minima of $r$ and gather the consecutive intervals together until both properties are verified. It is the principle used in the next algorithm:

**Fine to Coarse (FTC) Segmentation Algorithm:**

1. *Define the finest segmentation (i.e. the list of all the minima)* $S = \{s_1, \dots, s_n\}$ *of the histogram.*
2. *Repeat:*
   *Choose* $i$ *randomly in* $[2, length(S) - 1]$. *If the modes on both sides of* $s_i$ *can be gathered in a single interval* $[s_{i-1}, s_{i+1}]$ *following the unimodal hypothesis, group them. Update* $S$.
   *Stop when no more unions of successive intervals follows the unimodal hypothesis.*
3. *Repeat step 2 with the unions of* $j$ *intervals,* $j$ *going from* $3$ *to* $length(S)$.

A result of this algorithm is shown on Figure 2. The left part of the figure shows the initialization of the algorithm (all the minima), and the final result is on the right.

Now that we have set up an algorithm which ensures the construction of an acceptable segmentation, we will devote the next section to the application of the proposed method to color image segmentation.

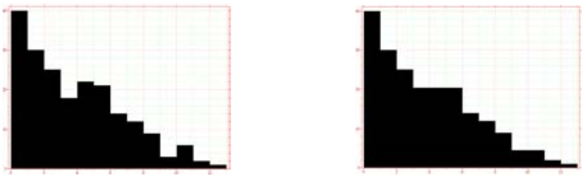

**Fig. 1.** The right histogram is the Grenander estimator of the left histogram, computed by the "Pool Adjacent Violators" algorithm. Observe that the new histogram is a decreasing function.

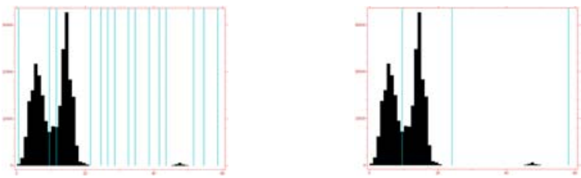

**Fig. 2.** Left: all the minima of the histogram. Right: remaining minima after the fine to coarse algorithm.

## 3 Color Image Segmentation

We apply the FTC algorithm on the hue histogram of a color image, in order to obtain a first segmentation. At this step a lot of color information, contained in the saturation and intensity components, has been lost. Then, we follow the same process by applying the algorithm to the saturation and intensity histograms of each mode obtained previously.

For the practical implementation of the algorithm, we must take into account that, in the discrete case, a quantization problem appears when we try to assign hue values to quantized color points in the neighborhood of the grey axis. A solution to this problem is to discard points that have saturation smaller than $\frac{Q}{2\pi}$, where $Q$ is the number of quantized hue values. This requirement defines a cylinder in the HSI color space called the *grey cylinder*, since all the points contained in it will be considered as grey values.

Our algorithm can be described by the following steps:

**Color Image Segmentation Algorithm**

1. *Apply the FTC algorithm on the hue histogram of the image. Let $S$ be the obtained segmentation.*
2. *Link each pixel of the grey cylinder to its corresponding interval $S_i = [s_i, s_{i+1}]$, according to its hue value.*
3. *For each $i$, construct the saturation histogram of all the pixels of the image whose hue belongs to $S_i$. Do not take into account the pixels of the grey cylinder. Apply the FTC algorithm on the corresponding saturation histogram. For each $i$, let $\{S_{i,i_1}, S_{i,i_2}, \ldots\}$ be the obtained segmentation.*
4. *For each $i$, link each pixel of the grey cylinder which belonged to the interval $S_i$, to the lower saturation interval $S_{i,i1}$ obtained in the previous segmentation step.*
5. *For each $i$ and each $j$, compute and segment the intensity histogram of all the pixels whose hue and saturation belong to $S_i$ and $S_{i,i_j}$, including those of the grey cylinder.*

It is also worth noting that the hue histogram is circular, which means that the hue value $0°$ is identified to the hue value $360°$.

## 4 Results

For each experiment we show five images: the first one is the original image; the second one corresponds to the hue histogram with the obtained modes marked with a dashed line between them; the third one is the segmented image after the application of the algorithm to the hue histogram; the fourth one corresponds to the segmented image after the application of the algorithm to the histograms of hue and saturation; and, finally, the segmented image after the application of the algorithm to the histograms of hue, saturation and intensity. The segmented images are displayed with the different modes represented by the mean values of the hue, saturation and intensity of all the pixels in the mode.

Figure 3 is the “ladybug” image in which we distinguish three different colors corresponding to different objects: background, leaf and ladybug. After applying the proposed separation approach to the hue histogram, we obtain three different modes, which correspond to the three referred objects. It is clear that the modes are detected independently of their relative number of pixels. Detection only depends on the meaningfulness of the mode, allowing the detection of small objects, as in the present example. The total number of colors in the final segmentation is 11, because of the great variety of shades in the background.

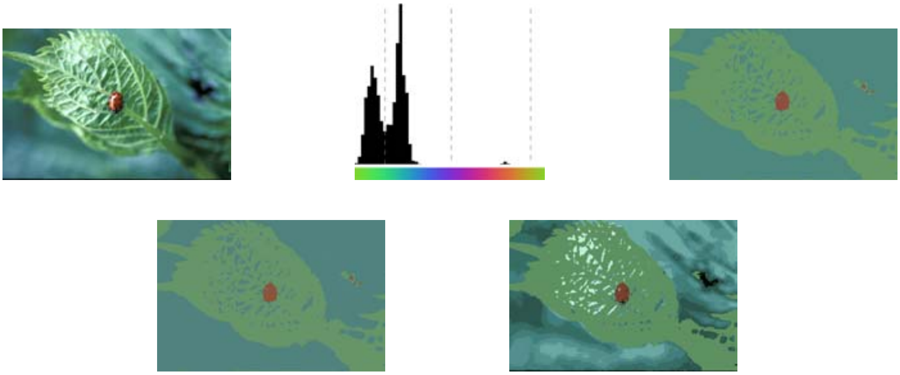

**Fig. 3.** The results of the proposed approach: Original image “ladybug”; hue histogram with the three obtained modes; resulting image with 3 colors after hue segmentation; resulting image with 4 colors after hue and saturation segmentation; resulting image with 11 colors after hue, saturation and intensity segmentation.

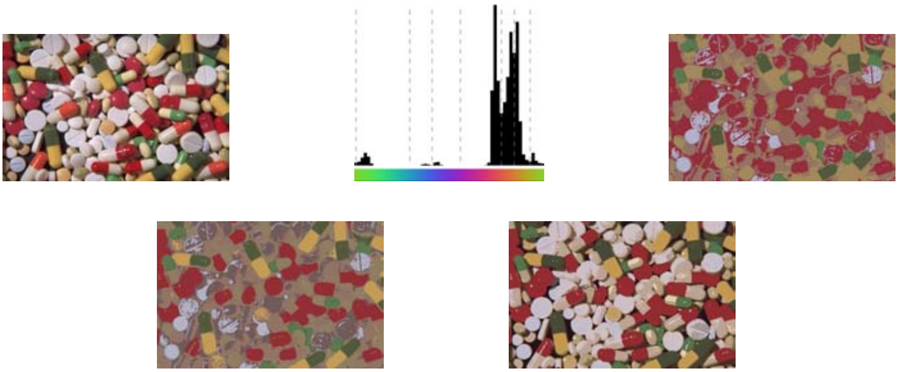

**Fig. 4.** The results of the proposed approach: Original image “pills”; hue histogram with the seven obtained modes; resulting image with 7 colors after hue segmentation; resulting image with 9 colors after hue and saturation segmentation; resulting image with 23 colors after hue, saturation and intensity segmentation.

In figure 4 we obtain seven modes in the first step of the segmentation. These colors correspond to two different greens, two different brows, one red, one blue and one cyan, which are clearly distinguished in the original image. Then, the saturation step add two new colors and finally we obtain twenty-three colors.

Figure 5 displays a third experiment. See figure caption for details.

## 5 Conclusion

In this paper, we have presented a new histogram thresholding method for color image segmentation. The method searches the optimal separators in the hue, saturation and intensity histograms of the color image. As a result, we obtain different modes which correspond to different regions in the image. This permits to segment the color image, by representing the different regions by their mean color.

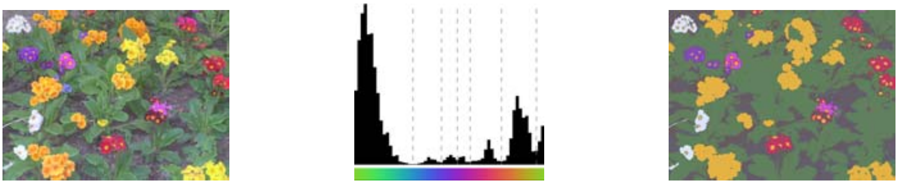

**Fig. 5.** The results of the proposed approach: Original image “flowers”; hue histogram with the six obtained modes; resulting image with 11 colors after hue, saturation and intensity segmentation.

As an application of the proposed method, we aim at a color analysis algorithm giving automatically, for every color image, a small, consistent and accurate list of the names of colors present in it, thus yielding an automatic color recognition system.

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